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Reduced Differential Transform Method (RTDM) for the Wave Equation

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Abstract: **The paper examines the structure and applications of reduced differential transform method (RTDM). This calculation and transformation are used to solve, model, and illustrate complex physical and chemical phenomena and processes. It is considered as easier as and more convenient than other modelling methods. The paper applied RTDM methods to solve three wave examples. The problem is structured, boundary and initial conditions are framed, and the detailed formulas with calculations are provided. Researchers can use these examples in their studies.**

Keywords: **reduced differential transform method (RTDM), chemical phenomena, wave examples.**

I. RTDM BASICS

RTDM is used to develop physical and mathematical models that illustrate complex and multivariable phenomena. RTDM is used in the construction of non-linear partial differential equations. It is an iterative process that is used to obtain solutions for many physical and chemical processes such as inverse scattering, Hirota's bilinear model, Painleve expansion, homogenous methods, Backlund transformation, and others . The advantage of this method is that it reduces the number of computations and can be used to solve non-linear physical problems. When compared to other methods such as Adomian decomposition and Laplace decomposition, RTDM is relatively simple, and the number of steps involved is less. The equations that depict waves are classified as a second order, linear-partial-differential. It helps to understand water waves, heat waves, seismic waves, or light waves. This paper uses RTDM to solve a wave equation. Several definitions are available for RTDMs. This section first defines an RTDM function and then solves two well-known problems. For the definition, consider $u(x,t)$. This represents two variables u(x, t), and this is obtained when the two functions $u(x,t) = f(x)g(t)$ are multiplied.

Using the differential transform function u(x ,t) is now defined as:

$$u(x,t) = \sum_{i=0}^{\infty} F(i)x^i \sum_{j=0}^{\infty} G(j)t^j = \sum_{k=0}^{\infty} U_k(x)t^k$$

In the above, the spectrum of t-dimension of $u(x,t)$ is $U_k(x)$.

Another definition considered when $u(x,t)$ is of analytic nature, and differentiation is carried out for time *t*, space *x*.

$$U_k(x) = \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k}u(x,t)\right]_{t=0} , \qquad (1)$$

Where: The transformed function is $U_k(x)$ refers to the transformed function.

In (1), the transformed function is $u_k(x)$. The initial function is *u (x, t)* is the original function.

The differential transform inverse of $U_k(x)$ is given as:

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k \quad , \tag{2}$$

When equations (1) and (2) are combined, the expression formed is:

$$U(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!} \left[\frac{\partial^k}{\partial t^k} u(x,t) \right]_{t=0} t^k \tag{3}$$

The two definitions are used to define the RTFD. For power expression e, $u\ (x,\ t) = e^{x+t}$ this function is:

$$u(x,t) = e^{x+t} = \left(1 + x + \frac{x^2}{2} + \ldots\right)\left(1 + t + \frac{t^2}{2} + \ldots\right) = \sum_{i=0}^{\infty} F(i) x^i \sum_{j=0}^{\infty} G(j) t^j$$

Important ideas of RDTM are given for the general wave equation given below:

$$Lu(x,\ t) + Ru(x,\ t) + Nu\ (x,\ t) = g(x,\ t)$$

For initial conditions $u(x,0) = f(x)$

Where $L = \frac{\partial}{\partial t}$, $R = \frac{\partial^2}{\partial x^2}$ are the linear operators. $g(x,\ t)$ is an inhomogeneous term and $Nu(x,\ t)$ is a non-linear term. Using RTDN, the equation developed by using iteration steps is:

$$(k+1)U_{k+1}(x) = G_k(x) - RU_k(x) - NU_k(x) \quad , \tag{4}$$

Where $U_k(x), RU_k(x), NU_k(x)\ and\ G_k(x)$ are the transformations obtained for the functions

$Lu(x,t), Ru(x,t), Nu(x,t)\ and\ g(x,t)$.

The initial condition now is: $U_0(x) = f(x)$, (5)

When equation (5) is substituted into (4), and by using iterative calculation, values for $U_k(x)$ are obtained. The inverse transformation gives the solution below:

$$\widetilde{u_n}(x,t) = \sum_{k=0}^{\infty} U_k(x) t^k$$

Where n is the approximation solution order, and the equation formed is:

$$u(x,t) = \lim_{n\to\infty} \widetilde{u_n}(x,t)\ , \tag{6}$$

Based on the above expressions, a few examples to solve wave equations are given as follows.

II. EXAMPLE

1. Homogenous Wave Equation

The following homogenous equation is considered

$$u_{tt} = u_{xx} - 3u\ , \qquad 0 < x < \pi, \qquad t > 0 \tag{7}$$

Given the initial conditions:

$$u(x,0) = 0\ , \qquad u(x,0) = 2\cos(x)$$

Boundary conditions as: (8)

$$u(0,t) = \sin(2t)\ , \qquad u(\pi,t) = -sin(2t)$$

With function of the variables x and t as $u = u(x,t)$.

The base properties and formulas of RTDM are used to calculate the transform of (7), and it is given as:

$$\frac{(k+2)!}{k!}U_{k+2}(x) = \frac{\partial^2}{\partial x^2}U_k(x) - 3U_k(x)\,, \tag{9}$$

For the initial condition in (8), the expression is:

$$U_0(x) = 0, U_1(x) = 2\cos(x) \tag{10}$$

When (10) is placed in (9), the expression for $U_k(x)$ is obtained successively:

$$U_2(x) = 0$$

The equation reduces to:

$$U_3(x) = -\frac{4}{3}\cos(x)\,, U_4(x) = 0, U_5(x) = \frac{4}{155}\cos(x)\ , U_6(x) = 0, U_7(x) = -\frac{8}{315}\cos(x), \dots, U_k(x) = \frac{(-1)^{\frac{k-1}{2}}}{k!}2^k\cos(x)$$

for $U_k(x)$, the equation for differential inverse transform:

$$U(x) = \cos(x)\sum_{k=1,3,\dots}^{\infty}\frac{(-1)^{\frac{k-1}{2}}}{k!}2^k\,t^k\,, \tag{11}$$

The closed form of above equation is given as $u(x,t) = \cos(x)\sin(2t)$.

2. Inhomogeneous Non-Linear Wave Equation

Consider the inhomogeneous non-linear wave equation given as follows

$$u_{tt} = u_{xx} + u + u^2 - xt - x^2t^2\,, \qquad 0 < x < \pi, \qquad t > 0 \tag{12}$$

The boundary conditions:

B.C. $u(0,t) = 0,\ u(\pi,t) = \pi t$

Initial conditions (13)

I.C. $u(x,0) = 0\,,\ u_t(x,0) = x$

For the differential transform of equation (12) and the IC, we have:

$$\frac{(k+2)!}{k!}U_{k+2}(x) = \frac{\partial^2}{\partial x^2}U_k(x) + U_k(x) + \sum_{r=0}^{k}U_r(x)U_{k-r}(x) - x\delta(k-1) - x^2\delta(k-2), \tag{14}$$

The transformed initial condition

$$U_0(x) = 0, U_1(x) = x \quad , \tag{15}$$

When (15) is substituted in (14), we have:

$$U_2(x) = 0\,, U_3(x) = 0\ \ and\ U_k(x) = 0\,, for\ k = 4,5,6,\dots$$

The differential inverse transform is

$$u(x,t) = \sum_{k=0}^{\infty}U_k(x)t^k = xt$$

This is the required solution.

3. Wave Equation in Unbounded Domain

The last equation considered is for the unbounded domain equation. Please refer to the following equation

$$u_{tt} = u_{xx}\,, \qquad -\infty < x < \infty, \qquad t > 0 \qquad (16)$$

Initial conditions are I.C.

$$u(x,0) = \sin(x)\,, \qquad u_t(x,0) = 0 \qquad (17)$$

This is the solution for $u(x,t) = \sin(x)\cos(x)$.

Considering the initial condition and the differential transform, the solution is:

$$\frac{(k+2)!}{k!} U_{k+2}(x) = \frac{\partial^2}{\partial x^2} U_k(x), \qquad (18)$$

The I.C. that is now transformed is: $U_0(x) = \sin(x)\,, U_1(x) = 0$

By substituting the above two equation, we get:

$$U_2(x) = -\frac{1}{2}\sin(x)\,,\ U_3(x) = 0,\ U_4(x) = \frac{1}{24}\sin(x),\ \ U_5(x) = 0, U_6(x) = \frac{1}{7202}\sin(x)\,, \ldots.$$

This sequence can be continued for $U_k(x)$. When all values of $U_k(x)$ are substituted, then:

$$u(x,t) \approx \sum_{k=0}^{6} U_k(x)t^k = \sin(x) = -\frac{1}{2}\sin(x)t^2 + \frac{1}{24}\sin(x)t^4 - \frac{1}{7202}\sin(x)\,t^6$$

When a larger number of iterations are used, then $u(x,t)$ tends to converge to $\sin(x)\cos(t)$. This is the required solution.

It is possible to plot the RDTM calculation of eight power for the $u(x,t) = \sin(x)\cos(t)$.

III. CONCLUSION

The paper discussed the use of RTDM to solve wave equations. RTDM allows a fast and easy method of modelling the equation and reduces the computational work. Three examples of the application of RTDM to solve wave problems were studied. The examples discussed include homogenous equations, inhomogeneous equations, and for unbounded domain. For all the three examples, the basic problem was formulated with the boundary and initial conditions, and subsequent formulas were derived to solve the problems. The methods can be used in a wide range of physical and chemical processes and events.

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